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# Distorted Time Preferences and Time-to-Build in the Transition to a Low-Carbon Energy Industry

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**Abstract** We study the welfare-theoretic consequences of diverging social and private time-preference rates and time-to-build for the transition to a low-carbon energy industry. We show that time-to-build, a prevalent characteristic of capital accumulation in the energy sector, amplifies the distortion induced by the split discount rates. Thus, these two characteristics create in a mutually reinforcing way less favorable circumstances for the introduction of new clean energy technologies as compared to the social optimum, even if welfare losses from emissions are internalized. We discuss resulting policy implications with particular emphasis on the energy sector.

**Keywords** Discounting · Distorted time preferences · Energy industry · Environmental and technology policy · Technological transition · Time-to-build

**JEL Classification** Q48 · H23 · H43

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## 1 Introduction

How to accomplish the transition to a low-carbon energy industry in a socially optimal way is the subject of an ongoing debate. Typically, investments in the energy sector concern long-lived and cost-intensive capital goods associated, moreover, with particularly long construction times. At the same time, due to their environmental impact, such investments have long-term consequences for society as a whole. Another distinctive feature of the energy sector is its recent liberalization in most industrialized countries. Investment decisions are now mainly governed by private actors. As a consequence, not only environmental preferences, but also time preferences, both on social and on private levels, are likely to play a key role in the transition to a low-emission energy industry.

We investigate the transition from an established to a new energy technology in a stylized general equilibrium framework, which incorporates the distinctive features of the energy sector as outlined above. The *established* technology gives rise to an environmentally harmful pollutant, which can partly be disarmed by abatement effort. The *new* technology is clean, but its specific capital needs a positive time span to be built. That is, there is a time lag between the cost of investment and the new capital to become productive. We assume that in the status quo the established production technique is fully developed, while the new technology is only to be produced and, thus, may eventually replace the established one. In addition, we account for the recent liberalization of energy markets by assuming that investments in capital are governed by private actors who exhibit a private time preference rate which exceeds the social rate. We show that these two characteristics, the time-to-build feature and the split in time preference rates, create, in a mutually reinforcing way, less favorable circumstances for the introduction of the new and the replacement of the old energy technology compared to the social optimum, even if the welfare losses from emissions are fully internalized. We show how the social optimum can be achieved in a decentralized market economy by a combination of environmental and technology policies.

As an outcome of the discounting debate, it is well recognized in economics and finance that, in general, social and private time-preference rates differ for several reasons (e.g., Arrow and Lind 1970; Blanchard 1985; Drèze and Stern 1990; Frederick et al. 2002; Groom et al. 2005; Mehra and Prescott 2003; Portney and Weyant 1999; Stiglitz 1982; Tirole 1981; Yaari 1965). Traditional categories include distortionary taxation, distortionary public policies, imperfect competition and production externalities, the regulation of which is generally clear, at least in theory. More recent contributions stress agency issues, for example in form of the ‘short-termism’ to which managers are urged by their contracts (e.g., DeMarzo et al. 2009). Moreover, the *de-facto* unavailability of private bonds with arbitrary maturities may be welfare-deteriorating when bond yield curves decline for long maturities (Gollier 2002, 2010).

In view of the ongoing discussion with respect to the causes of split time preferences, we omit an endogenous explanation of the time-preference distortion. For analytical tractability, we stick to the most simple case of a world where outcomes are certain and preferences are separable in time and in consumption of a final good and emissions. This allows us to analyze the welfare implications of split time-preference rates for time-lagged technological transitions in general and to treat policy implications of those cases, where the market failure underlying the distortion cannot otherwise be remedied.

Our paper complements the wide-spanned literature on induced technological change and the environment. In this literature, the intertemporal nature of the climate change problem is mostly addressed either in endogenous growth or integrated assessment models. Top-down approaches study induced technological change by applying one representative aggregated

production technology, which becomes more efficient and/or less polluting by technological change (e.g., Bovenberg and Smulders 1995; Goulder and Mathai 2000; Nordhaus 2002; Newell et al. 1999; Tahvonen and Salo 2001). In bottom-up approaches, induced technological change also allows for structural change between competing technologies (e.g., Gerlagh and Van der Zwaan 2003; Goulder and Schneider 1999; Van der Zwaan et al. 2002). The two kinds of approaches commonly model technological change endogenously as a gradual improvement resulting either from R&D investments or learning by doing. They focus on positive spillovers to other firms from the innovation process or dynamic increasing returns stemming from learning by using, learning by doing or network externalities typically related to the diffusion of new technologies as sources of market failure inducing technology policy (Jaffe et al. 2005). In contrast to this literature, we abstract from these components of long-run technological change in particular, and growth in general. Instead we emphasize the welfare-theoretic consequences of the split of social and private time preference rates and the time-to-build feature in a framework of structural change. As in Winkler (2008), we rather adopt a medium-term perspective, in which the set of available technologies is given, and the system dynamics is governed by the accumulation of the corresponding specific capital stocks.<sup>1</sup>

Although derived from a stylized theoretical model, our results have direct policy implications for the energy sector in particular, and technological transitions in general. With respect to the former, we expect the energy sector, due to its long construction times and the implied reinforcing effect, to be particularly vulnerable for inefficiencies caused by split time-preference rates. In addition, our results give new theoretical support for subsidizing new less polluting energy technologies. From a more general perspective, we provide a new reason why environmental regulation should be complemented by technology policy.

The paper is organized as follows. The model is introduced in Sect. 2. In Sects. 3 and 4, we solve the intertemporal optimization problems in the social optimum and in the decentralized competitive market economy, and derive conditions for partial and full replacement of the established by the new energy technology. In Sect. 5, we discuss model assumptions and policy implications. Section 6 concludes.

## 2 The Model

Consider an economy composed of two vertically integrated sectors, the energy sector and the investment sector. Labor constitutes the only primary input. It is by assumption fixed to unity at all times  $t$ . The *energy* sector comprises two technologies, an *established* and a *new* one. The established technology is fully set up at the beginning of the planning horizon. As a consequence, we do not explicitly consider capital for the established technology. We include the costs of employing and maintaining the capital stock into the labor costs which are normalized to 1. The established technology generates one unit of energy  $x$  for every unit of labor  $l_1$  employed. In addition, each unit of output produced gives rise to one unit of an unwanted and harmful joint output  $j$ :

$$x_1(t) = l_1(t) = j(t). \quad (1)$$

<sup>1</sup> Formally, our paper builds on the structural change frameworks of Winkler (2005) and Winkler et al. (2005), and is more loosely related to recent growth models with time-lagged stock accumulation (e.g., Bambi 2008; Boucekine et al. 2005; Fabbri and Gozzi 2008).

Abatement effort  $a$  per unit of energy (partially) reduces the joint output. The function  $G$  denotes the fraction of the joint output  $j$  which is disarmed by abatement.  $G$  is assumed to be twice continuously differentiable, satisfying  $G(0) = 0$ ,  $G' > 0$ ,  $G'' < 0$  and  $\lim_{a \rightarrow 1} G(a) = 1$ . We impose Inada conditions  $\lim_{a \rightarrow 0} G'(a) = \infty$ ,  $\lim_{a \rightarrow 1} G'(a) = 0$  to ensure that the abatement effort  $a$  is strictly positive and finite along the optimal path as long as  $l_1 > 0$ . Then, net emissions  $e$  equal the amount of joint output  $j$  minus abatement:

$$e(t) = x_1(t) (1 - G(a(t))). \quad (2)$$

The new technology employs  $\lambda$  units of labor together with one unit of the specific capital good  $k$  to produce one unit of energy:

$$x_2(t) = \min \left( \frac{l_2(t)}{\lambda}, k(t) \right). \quad (3)$$

Without loss of generality, the new technology does not produce an unwanted joint output. Energy is assumed to be homogeneous, such that total production  $x$  equals:

$$x(t) = x_1(t) + x_2(t). \quad (4)$$

The *investment* sector employs one unit of labor to produce one unit of the capital good. We assume that the creation of new capital goods needs a positive time span  $\sigma$ . That is, there is a time lag  $\sigma$  between the costs of investment  $i$  and the emergence of productive capital  $k$ . The intuition behind this assumption is twofold. On the one hand, power plants are not built in a day but need *substantial time* for creation.<sup>2</sup> On the other hand, the time lag  $\sigma$  can also be identified with the time required for the R&D of a new technology. In addition, the capital stock  $k$  deteriorates at the constant and exogenously given rate  $\gamma$ , implying the following equation of motion:

$$\dot{k}(t) = i(t - \sigma) - \gamma k(t), \quad \gamma > 0. \quad (5)$$

Due to the time lag  $\sigma$  the equation of motion for the capital stock (5) constitutes a *retarded differential-difference* equation. Thus, variations of the capital stock  $k$  do not only depend on parameters evaluated at time  $t$  but also on parameters evaluated at the earlier time  $t - \sigma$ .

The labor constraint implies that

$$1 \geq [1 + a(t)] l_1(t) + l_2(t) + i(t), \quad (6)$$

holds at all times  $t$ .<sup>3</sup> Assuming efficient labor allocation among the three production processes, i.e.,  $1 = [1 + a(t)] l_1(t) + l_2(t) + i(t) \forall t$ , and full employment of the capital stock, i.e.,  $x_2(t) = l_2(t)/\lambda = k(t)$ , we obtain the following formulae for total energy  $x(t)$  and net emissions  $e(t)$ :

$$x(t) = \frac{1 - \lambda k(t) - i(t)}{1 + a(t)} + k(t), \quad (7a)$$

$$e(t) = [1 - G(a(t))] \frac{1 - \lambda k(t) - i(t)}{1 + a(t)}. \quad (7b)$$

<sup>2</sup> In general, the time span  $\sigma$  strongly depends on the type of technology. While a nuclear power plant may take five to seven years to be built, a gas co-generation plant is set up in a year or two.

<sup>3</sup> Note that we have defined abatement effort  $a$  per unit of energy produced via the established energy technology. Thus, total abatement effort equals  $a(t)x_1(t) = a(t)l_1(t)$ .

To close the model we consider a representative consumer who derives instantaneous utility from consumption of the final product and disutility from net emissions.<sup>4</sup> Like Arrow and Kurz (1970, 116) we assume that the representative consumer's private rate of time preference differs from the social. That is, the representative consumer applies different *intertemporal* weights between welfare today and welfare tomorrow compared to a social planner maximizing social welfare. For simplicity, we consider instantaneous welfare to be additively separable in energy consumption  $x$  and net emissions  $e$ . As a consequence, the representative consumer (privately) maximizes

$$W_p = \int_0^{\infty} [U(x(t)) - D(e(t))] \exp[-\rho_p t] dt, \quad (8a)$$

whereas, at the same time, the social planner maximizes

$$W = \int_0^{\infty} [U(x(t)) - D(e(t))] \exp[-\rho t] dt, \quad (8b)$$

where  $U$  and  $D$  are twice differentiable functions with  $U' > 0$ ,  $U'' < 0$ ,  $\lim_{x \rightarrow 0} U' = \infty$  and  $D'(0) \geq 0$ ,  $D' > 0$  for any positive amount of emissions  $e$ , and  $D'' > 0$ . We concentrate on the empirically relevant case that the private rate of time preference  $\rho_p$  exceeds the socially efficient rate  $\rho$ , i.e.,  $\rho_p > \rho$ . That is, individual actors are in a private decision context more impatient to consume than society as a whole.

We want to emphasize that the split in time-preference rates does *not* imply that the representative consumer and the social planner have preferences that differ in an arbitrary way. On the contrary, we impose that both exhibit *essentially* the same preferences, as expressed in the instantaneous utility function  $U(x(t)) - D(e(t))$ . However, we acknowledge that certain circumstances may cause a split in the time-preference rates. Apart from the causes already mentioned in Sect. 1, we want to give two further intuitive examples. First, individuals only live for a finite time. As a consequence, they ask for a higher interest rate than an infinitely lived individual, as they face the risk not to survive the repayments. If, for example, individuals are risk-neutral, exhibit a pure time preference rate  $\rho$  and face a Poisson-distributed death probability  $p$ , the resulting *effective* rate of time-preference is  $\rho + p$  (Blanchard 1985, see also Yaari 1965). Identifying the representative consumer with the finitely lived private households and the social planner with the state as an eternal entity leads to  $\rho_p > \rho$ .<sup>5</sup> Second, a state as a large investor may have the chance to better diversify and, therefore, better ensure itself against risk than an individual private investor. Everything else equal, the private investor would, thus, ask for a higher return on investment than the social planner. In our stylized model this can be modelled by assuming a higher time-preference rate of the representative consumer. However, due to the variety of different causes for a split in the time-preference rate, we neither model finite lifetimes or risk diversification explicitly, but assume split time-preference rates as a stylized fact and rather focus on the *implications* of this assumption.

<sup>4</sup> Obviously, CO<sub>2</sub> is a stock and not a flow pollutant. However, assuming that the negative externality on utility is caused by the emissions and not the global stock simplifies further calculations without impacting on our qualitative results (for further discussion, see Sect. 5).

<sup>5</sup> Calvo and Obstfeld (1988) show in a growth model with different generations of consumers with uncertain finite lifetimes that the optimal long-run interest rate corresponds to the social planner's generational discount rate, but need not coincide with the individuals' subjective utility discount rates.

### 3 Social Optimum

We now derive the optimal plan for the development of the model economy. As outlined in Sect. 2, social welfare is given by Eq. (8b). Thus, the social planner solves the following maximization problem:

$$\max_{a(t), i(t)} \int_0^{\infty} [U(x(t)) - D(e(t))] \exp[-\rho t] dt, \quad (9a)$$

subject to Eqs. (5), (7a), (7b), the inequality constraints

$$0 \leq i(t) \leq 1 - \lambda k(t), \quad (9b)$$

and the initial conditions

$$k(0) = 0 \quad i(t) = 0, \quad t \in [-\sigma, 0]. \quad (9c)$$

For the dynamics of the economy it is important that, due to the linearity of the production techniques, two *corner solutions* can occur along the optimal path. It may be optimal not to invest in the new technology, which corresponds to  $i(t) = 0$ . Or, it may be optimal only to use the new technology, in which case all labor is utilized to employ and maintain the capital stock, i.e.,  $i(t) = 1 - \lambda k(t)$  and  $I_1(t) = a(t) = 0$ . As a consequence, we have to explicitly check for these two corner solutions to characterize the complete dynamics of the model economy.

#### 3.1 Necessary and Sufficient Conditions for the Social Optimum

To solve the optimization problem of the social planner we apply the generalized maximum principle derived in El-Hodiri et al. (1972) for time-lagged optimal control problems. One obtains the following present-value Hamiltonian  $\mathcal{H}$ :

$$\begin{aligned} \mathcal{H} = & [U(x(t)) - D(e(t))] \exp[-\rho t] + q_k(t + \sigma) i(t) - q_k(t) \gamma k(t) \\ & + q_x(t) \left[ \frac{1 - \lambda k(t) - i(t)}{1 + a(t)} + k(t) - x(t) \right] + q_{\bar{t}}(t) i(t) \\ & + q_e(t) \left[ (1 - G(a(t))) \frac{1 - \lambda k(t) - i(t)}{1 + a(t)} - e(t) \right] + q_{\bar{t}}(t) [1 - \lambda k(t) - i(t)], \quad (10) \end{aligned}$$

where  $q_k$  denotes the costate variable or shadow price of the capital stock  $k$ , and  $q_x, q_e, q_{\bar{t}}$  and  $q_{\bar{t}}$  denote the Kuhn-Tucker parameters for the (in)equality conditions (7a), (7b) and (9b).

Assuming the Hamiltonian  $\mathcal{H}$  to be continuously differentiable with respect to the control variables  $a$  and  $i$ , the following necessary conditions hold for an optimal solution:

$$q_x(t) = U'(x(t)) \exp[-\rho t], \quad (11a)$$

$$q_e(t) = -D'(e(t)) \exp[-\rho t], \quad (11b)$$

$$0 = \frac{1 - \lambda k(t) - i(t)}{[1 + a(t)]^2} \{q_x(t) + q_e(t) [1 - G(a(t)) + (1 + a(t))G'(a(t))]\}, \quad (11c)$$

$$q_k(t + \sigma) + q_{\bar{i}}(t) = \frac{q_x(t)}{1 + a(t)} + q_e(t) \frac{1 - G(a(t))}{1 + a(t)} + q_{\bar{i}}(t), \quad (11d)$$

$$\dot{q}_k(t) = \gamma q_k(t) - q_x(t) \frac{1 + a(t) - \lambda}{1 + a(t)} + \lambda q_e(t) \frac{1 - G(a(t))}{1 + a(t)} + \lambda q_{\bar{i}}(t), \quad (11e)$$

$$q_{\bar{i}}(t) \geq 0, \quad q_{\bar{i}}(t)i(t) = 0, \quad (11f)$$

$$q_{\bar{i}}(t) \geq 0, \quad q_{\bar{i}}(t) [1 - \lambda k(t) - i(t)] = 0. \quad (11g)$$

As the Hamiltonian is strictly concave along the optimal path (see Appendix A.1), the optimal solution is unique and the necessary conditions (11a)–(11g) are also sufficient if, in addition, the following transversality condition holds:

$$\lim_{t \rightarrow \infty} q_k(t)k(t) = 0. \quad (11h)$$

Conditions (11a) and (11b) state that along the optimal path the shadow price of energy equals the marginal utility of energy and the shadow price of net emissions equals the marginal disutility of net emissions. By inserting conditions (11a) and (11b) in condition (11c), we obtain for  $i(t) < 1 - \lambda k(t)$ :

$$U'(x(t)) = D'(e(t)) [G'(a(t)) (1 + a(t)) + 1 - G(a(t))]. \quad (12)$$

This condition expresses that along the optimal path (and for  $i(t) < 1 - \lambda k(t)$ ) the utility of an additional marginal unit of energy equals the disutility of the emissions that it induces. Along the optimal path this equation determines the optimal value of the abatement effort  $a$  per unit of output  $x_1$ . If  $i(t) = 1 - \lambda k(t)$ , implying that labor input in the established production technology  $l_1$  and abatement effort  $a$  equal zero, condition (11c) reduces to the truism  $0 = 0$ .

Together with the transversality condition (11h), and inserting conditions (11a) and (11b), condition (11e) can be unambiguously solved to yield:

$$\begin{aligned} q_k(t) = & \exp[-\rho t] \int_t^\infty \frac{U'(x(t'))(1 + a(t') - \lambda) + \lambda [D'(e(t')) (1 - G(a(t')) - q_{\bar{i}}(t'))]}{1 + a(t')} \\ & \times \exp[-(\gamma + \rho)(t' - t)] dt'. \end{aligned} \quad (13)$$

Along the optimal path the shadow price for the capital stock equals the net present value of all future welfare gains of one additional marginal unit of the capital good. As capital goods are long-lived, they contribute over the whole time horizon (increasingly less though due to deterioration and discounting). The fraction under the integral equals the marginal instantaneous welfare gain of an additional unit of capital, which comprises two components. The first is the direct welfare gain due to the energy produced. It is positive if the new technology needs less labor input per unit of output than the established one, i.e.,  $\lambda < 1 + a(t)$ . The second term is always positive and denotes the welfare gain due to emissions abated by switching from the established to the new production technique.

Inserting conditions (11a) and (11b) in Eq. (11d) yields:

$$q_k(t + \sigma) + q_i(t) = \frac{\exp[-\rho t]}{1 + a(t)} [U'(x(t)) - D'(e(t))(1 - G(a(t)))] + q_i(t). \quad (14)$$

The equation states that along the optimal path, and as long as the inequality constraints (9b) are not binding, the present value of the welfare loss by investing in one marginal unit of new capital, which is given by the present value welfare gain of the alternative use of one marginal unit of labor in the established production technique minus the resulting disutility from emissions (right-hand side), equals the net present value of the sum of all future welfare gains by using the new capital good in production (left-hand side). As the investment needs the time span  $\sigma$  to become productive capital, the sum of all future welfare gains of an investment at time  $t$  is given by the shadow price of capital at time  $t + \sigma$ ,  $q_k(t + \sigma)$ . If it is not optimal to invest, i.e.,  $q_i(t) \geq 0$ , the future welfare gains of an investment are weakly smaller than the costs of investment. Further, if all labor is used to employ and maintain the capital stock, i.e.,  $q_i(t) \geq 0$ , the future welfare gains of an investment weakly exceed the costs of investment.

As noted above, the optimal system dynamics of the optimization problem (9) splits into three qualitatively different regimes. These different regimes are determined by whether and which of the two inequality constraints (9b) are binding:

1. No investment:

The new technology may be so inferior compared to the established technology that it is not used in the long run. In fact, if it is not optimal to invest in the new technology in  $t = 0$ , then it is optimal to *never invest* in the new technology.<sup>6</sup> In this case, in which the inequality constraint  $i(t) \geq 0$  is binding at all times, the economy does not exhibit any transitional dynamics but instantly switches to and stays in the corner solution  $i(t) = 0$ ,  $k(t) = 0$ ,  $x_1(t) = x_1^0 > 0$ ,  $a(t) = a^0 > 0$ ,  $e(t) = e^0 > 0$  for all  $t$ .

2. Full replacement:

The new technology may be so superior compared to the established technology that it eventually fully replaces the established one. That is, from some time  $\bar{t}$  onwards the economy enters a regime in which all labor is solely used to employ and maintain the capital stock for the new technology. In this case, in which the inequality constraint  $i(t) \geq 1 - \lambda k(t)$  is binding for all  $t \geq \bar{t}$ , the economy either converges to a stationary state in which  $i(t) = i^\infty > 0$ ,  $k(t) = k^\infty > 0$ ,  $x_1(t) = a(t) = e(t) = 0$  or converges to a limit cycle around this stationary state.

3. Partial replacement:

Finally, the new technology may be such that it is optimal to invest in the new technology but the new technology is not superior enough compared to the established technology to fully replace it. In this case it is not entirely clear what the system dynamics looks like, as it is governed by a system of functional differential equations. However, the system exhibits a unique stationary state with  $i(t) = i^* > 0$ ,  $k(t) = k^* > 0$ ,  $x_1(t) = x_1^* > 0$ ,  $a(t) = a^* > 0$  and  $e(t) = e^* > 0$ . Fortunately, the system dynamics of the third regime is irrelevant to our analysis.

In Appendix A.2 we further discuss the system dynamics of the first and second regime, on which our results in the following are based.

<sup>6</sup> This holds because if no investment at  $t = 0$  is optimal, no capital is accumulated. As a consequence, the optimization problem at time  $t + \Delta t$  is identical to the optimization problem at time  $t = 0$ . As it was not optimal to invest at  $t = 0$  it is also not optimal to invest at time  $t + \Delta t$ .



### 3.2 Conditions for Investment and Replacement

Thus far, it is not clear which of the three possible regimes apply for a given economy. We now derive conditions which classify all possible economies into the three different regimes by their set of exogenous parameters. These conditions determine whether there is any investment in the new technology, and if so, whether the established technology is eventually fully replaced by the new one. We start with the no investment condition.

In order to derive a condition which identifies whether no investment is optimal, we assume that it is optimal never to invest, i.e., the economy stays in the corner solution  $i(t) = 0, \forall t$ . The following proposition states the condition for which this corner solution satisfies the necessary and sufficient condition for an optimal solution.

**Proposition 1** (No investment condition in the social optimum) *Given optimization problem (9), there is no investment in the new technology, i.e.,  $i(t) = 0 \forall t$  if and only if*

$$1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)} \leq \lambda + (\gamma + \rho) \exp[\rho\sigma], \quad (15)$$

where  $a^0$  is determined by the unique solution of the implicit equation:

$$U'(1 - a^0) = D'((1 - a^0)(1 - G(a^0))) [1 - G(a^0) + (1 + a^0)G'(a^0)]. \quad (16)$$

The proof is given in Appendix A.3.

Condition (15) has an intuitive economic interpretation. The left-hand side corresponds to the unit costs of production of the established technology,  $UC_{T_1}^0$ , the right-hand side to the unit costs of production of the new technology,  $UC_{T_2}^0$ . Thus, condition (15) states that no investment in the new technology is optimal if its unit costs of production are greater or equal to those of the established technology, i.e.,  $UC_{T_2}^0 \geq UC_{T_1}^0$ .

In the centralized economy,  $UC_{T_1}^0$  comprises three components, the ‘pure’ labor costs per unit of energy production, the labor costs for abatement per unit, and the social costs of unit emissions in terms of labor.  $UC_{T_2}^0$  comprises, apart from the ‘pure’ labor costs, the costs for building up and maintaining the necessary capital good in terms of labor. Obviously, the capital costs per unit of output depend positively on the dynamic characteristics  $\gamma$  and  $\sigma$  of the capital good production, as well as on the time preference rate  $\rho$ . In particular, the longer the time lag  $\sigma$  and the higher the rate of time preference  $\rho$  the higher are the unit costs of production of the new technology.

Despite the infinite time horizon and the linearity of the two production techniques, a violation of condition (15) does not guarantee full replacement of the established technology by the new technology in the long run. In the following, we deduce conditions for which complete or partial replacement occur. Formally, full replacement of the established by the new production technique implies that the economy is in the  $i(t) = 1 - \lambda k(t)$  corner solution in the long run. The inference of a condition for full replacement is similar to that of Proposition 1. We investigate under which conditions a full replacement stationary state, in which all labor is used to employ and maintain the fully developed new technology, is consistent with the necessary and sufficient conditions for an optimal solution. Proposition 2 states the result.

**Proposition 2** (Full replacement condition in the social optimum) *Given optimization problem (9) and assuming  $U'(x^\infty) - D'(0) \neq 0$ , full replacement of the established technology*

by the new one in the long-run stationary state is consistent with the necessary and sufficient conditions for a social optimum, if and only if

$$1 + \frac{D'(0)}{U'(x^\infty) - D'(0)} \geq \lambda + (\gamma + \rho) \exp[\rho\sigma], \quad (17)$$

where  $x^\infty$  is given by  $x^\infty = \frac{1}{\lambda + \gamma}$ .

The proof is given in Appendix A.4.

Proposition 2 says that full replacement can only occur if the costs per unit of output of the new technology in the full replacement stationary state  $UC_{T_2}^\infty$  (right-hand side) are smaller than or equal to the costs of the established technology  $UC_{T_1}^\infty$  (left-hand side). As there are no emissions in the full replacement stationary state, abatement effort is zero and  $UC_{T_1}^\infty$  only consists of the ‘pure’ labor costs plus the social costs, which stem from the damage of the first marginal unit of emissions. In the common case that the first marginal unit of emissions does not induce any environmental damage, i.e.,  $D'(0) = 0$ ,  $UC_{T_1}^\infty$  reduces to the ‘pure’ labor costs of production.<sup>7</sup>

For full replacement to occur, condition (15) must be violated while at the same time condition (17) holds. A straightforward corollary from Propositions 1 and 2 is that the established technology is only *partially* replaced by the new one, if conditions (15) and (17) are simultaneously violated.

**Corollary 1** (Partial replacement condition in the social optimum) *Given optimization problem (9) and  $U'(x^\infty) - D'(0) \neq 0$ , partial replacement of the established technology by the new one is optimal in the long-run if and only if*

$$1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)} > \lambda + (\gamma + \rho) \exp[\rho\sigma] > 1 + \frac{D'(0)}{U'(x^\infty) - D'(0)}, \quad (18)$$

where  $x^\infty = \frac{1}{\lambda + \gamma}$  and  $a^0$  is given by the unique solution of the implicit equation (16).

In sum, investment is not optimal if and only if the labor costs per unit of output of the new technology,  $UC_{T_2} = UC_{T_2}^0 = UC_{T_2}^\infty$ , are higher than the labor costs per unit of output of the established technology in the no investment corner solution,  $UC_{T_1}^0$ . If it is optimal to invest, i.e.,  $UC_{T_2} < UC_{T_1}^0$ , full replacement is optimal in the long run if and only if, in addition,  $UC_{T_2} \leq UC_{T_1}^\infty$  holds. Otherwise,  $UC_{T_1}^\infty < UC_{T_2} < UC_{T_1}^0$ , and the new technology will partly replace the established technology in the long run.

## 4 Competitive Market Equilibrium

We now consider a decentralized economy, in which a representative household and two representative firms interact on competitive markets for labor, capital and energy, which are cleared at all times.<sup>8</sup> Due to the emission externality and the split time-preference rates, the long-run stationary state in the decentralized economy falls, in general, short of the

<sup>7</sup> Condition (17) is not well defined if  $\lim_{x \rightarrow x^\infty} U'(x) = D'(0)$ . However, full replacement will still occur if, in addition, condition (15) holds, as the welfare gain of an additional unit of labor assigned to the old technology vanishes while the shadow price of capital, which is the net present value of all future welfare gains of an additional unit of capital, remains positive.

<sup>8</sup> We present a decentralized market economy in which the households directly manufacture the capital good in “home production” by means of labor and rent it to the firms. An alternative market economy encompassing

social optimum. We show how the social optimum can be implemented by complementing a standard emission tax with an investment subsidy.

#### 4.1 Representative Household

We assume the household to own the two firms and the total labor and capital endowments of the economy. Thus, the household chooses between selling labor to the firms at the market price of labor  $w$  and investing labor in the accumulation of capital  $k$ , which the household rents to the firms at the market price of capital  $r$ . In addition, the household buys energy  $x$  and may profit from the investment subsidy  $\tau_i(t)$ , paid per unit of investment  $i$ . Choosing energy as numeraire, the following budget constraint has to hold at all times  $t$ <sup>9</sup>:

$$x(t) = w(t)(1 - i(t)) - \tau_i(t)i(t) + r(t)k(t) + \pi_1(t) + \pi_2(t), \quad (19)$$

where  $\pi_1$  and  $\pi_2$  denote the profits of firms 1 and 2. In addition, capital can be accumulated according to Eq. (5). Assuming that the representative household maximizes its intertemporal welfare, as given by Eq. (8a), implying that it applies a *higher* rate of time preference  $\rho_p$  in the decentralized market regime than in the social decision context, the household solves the following maximization problem:

$$\max_{i(t)} \int_0^{\infty} [U(x(t)) - D(e(t))] \exp[-\rho_p t] dt, \quad (20)$$

subject to Eqs. (5), (19) the inequality constraint

$$i(t) \geq 0, \quad (21)$$

and the initial conditions (9c).

Thus, the present value Hamiltonian  $\mathcal{H}^H$  reads:

$$\begin{aligned} \mathcal{H}^H = & [U(x(t)) - D(e(t))] \exp[-\rho_p t] + q_k(t + \sigma)i(t) - q_k(t)\gamma k(t) \\ & + q_b(t)[w(t)(1 - i(t)) - \tau_i(t)i(t) + r(t)k(t) - p(t)x(t)] + q_i(t)i(t), \end{aligned} \quad (22)$$

where  $q_k$  denotes the costate variable or shadow price of the capital stock  $k$ , and  $q_b$  and  $q_i$  denote the Kuhn-Tucker parameters for the (in)equality conditions (19) and (21). Again, the strict concavity of the Hamiltonian  $\mathcal{H}^H$  (at least along the optimal path) ensures a unique solution.

Assuming that the Hamiltonian  $\mathcal{H}^H$  is continuously differentiable with respect to the control variable  $i$ , the following necessary conditions hold for an optimal solution:

$$q_k(t + \sigma) = (w(t) + \tau_i(t)) U'(x(t)) \exp[-\rho_p t] - q_i(t), \quad (23a)$$

$$\dot{q}_k(t) = q_k(t)\gamma - r(t)U'(x(t)) \exp[-\rho_p t], \quad (23b)$$

$$q_i(t) \geq 0, \quad q_i(t)i(t) = 0. \quad (23c)$$

Due to the strict concavity of the Hamiltonian, the necessary conditions (23a)–(23c) are also sufficient if, in addition, a transversality condition analogous to condition (11h) holds.

Footnote 8 continued

a third firm which produces the capital good by means of labor is conceivable. As long as also the third firm operates under conditions of perfect competition this does not alter the market equilibrium. A formal proof is available on request.

<sup>9</sup> Following the standard notation,  $\tau > 0$  denotes a tax and  $\tau < 0$  corresponds to a subsidy.

Together with this transversality condition, condition (23b) can be unambiguously solved to yield:

$$q_k(t) = \exp[\gamma t] \int_t^{\infty} r(s) U'(x(t)) \exp[-(\gamma + \rho_p)s] ds. \quad (24)$$

## 4.2 Firms

Taking prices as given, the firms maximize their profits in the competitive market equilibrium. Firm 1 produces energy according to the first production technology, described by equations (1) and (7b). Given a tax  $\tau_e(t)$  per unit of emissions, its profit  $\pi_1$  at time  $t$  is given by:

$$\pi_1(t) = [1 - w(t)(1 + a(t)) - \tau_e(t)(1 - G(a(t)))] l_1(t). \quad (25)$$

Firm 1 chooses both labor  $l_1$  and abatement effort  $a$  such as to maximize the net present value of all future profits, which is equivalent to maximizing the profit  $\pi_1$  at all times  $t$ . A necessary condition for profit maximization is

$$\frac{\partial \pi_1(t)}{\partial a(t)} = [-w(t) + \tau_e(t)G'(a(t))] l_1(t) = 0, \quad (26)$$

which is an implicit equation for the unique optimal abatement effort  $a^*(t)$ , as long as  $l_1(t) > 0$  and  $\tau_e(t) > 0$ . If  $l_1(t) = 0$  or  $\tau_e(t) = 0$ , the optimal abatement effort  $a^*(t) = 0$ , as either no emissions have to be abated or emission abatement is a pure cost to the firm.

Profit function  $\pi_1(t)$  is linear in labor demand  $l_1(t)$ . Thus, the demand for  $l_1(t)$  is given by the following correspondence:

$$l_1(t) \begin{cases} = \infty, & \text{if } 1 > w(t)(1 + a(t)) + \tau_e(t)(1 - G(a(t))) \\ \in [0, \infty), & \text{if } 1 = w(t)(1 + a(t)) + \tau_e(t)(1 - G(a(t))), \\ = 0, & \text{if } 1 < w(t)(1 + a(t)) + \tau_e(t)(1 - G(a(t))) \end{cases} \quad (27)$$

where the optimal abatement effort  $a$  is given by the solution of the implicit equation  $\tau_e(t)G'(a(t)) = w(t)$  if  $l_1(t) > 0$ , and  $a(t) = 0$  if  $l_1(t) = 0$  or  $\tau_e(t) = 0$ .

Firm 2 produces energy according to the second production technology, described by Eq. (3). Neither the innovation subsidy  $\tau_i$  nor the emission tax  $\tau_e$  directly affects firm 2. Thus, the profit  $\pi_2$  at time  $t$  equals:

$$\pi_2(t) = [1 - \lambda w(t) - r(t)] k(t), \quad (28)$$

which is a linear function of  $k$ . As a consequence, the profit  $\pi_2$  is non-negative for any  $k > 0$ , as long as the value of outputs exceeds the value of inputs. Analogously to firm 1, firm 2 demands as much capital as possible together with  $\lambda k$  units of labor, if the value of the output exceeds the value of the inputs. Thus, the demand of firm 2 is given by the following correspondence:

$$k(t) \begin{cases} = \infty \wedge l_2(t) = \lambda k(t) = \infty, & \text{if } 1 > \lambda w(t) + r(t) \\ \in [0, \infty) \wedge l_2(t) = \lambda k(t), & \text{if } 1 = \lambda w(t) + r(t). \\ = 0 \wedge l_2(t) = 0, & \text{if } 1 < \lambda w(t) + r(t) \end{cases} \quad (29)$$

### 4.3 Necessary and Sufficient Condition for the Market Equilibrium

At the market equilibrium, all markets clear. Again, the market solution may exhibit two corner solutions, in which either the household never invests in capital, or the total labor endowment is used to employ and maintain the capital stock. In the former, firm 2 is unable to operate. In the latter, firm 1 is driven out of the market. First, we analyze the interior market equilibrium where both firms operate (i.e.,  $l_1(t), i(t) > 0$ ). From conditions (26), (27) and (29) we derive the following equations:

$$1 = \tau_e(t) [G'(a(t))(1 + a(t)) + 1 - G(a(t))], \quad (30)$$

$$w(t) = \frac{1 - \tau_e(t) (1 - G(a(t)))}{1 + a(t)}, \quad (31)$$

$$r(t) = \frac{1 + a(t) - \lambda + \lambda \tau_e(t) (1 - G(a(t)))}{1 + a(t)}. \quad (32)$$

Inserting Eq. (31) into Eq. (23a) yields

$$\left[ \frac{1 - \tau_e(t) (1 - G(a(t)))}{1 + a(t)} + \tau_i(t) \right] U'(x(t)) \exp[-\rho_p t] = q_k(t + \sigma), \quad (33)$$

which together with Eq. (30) determines the interior market equilibrium for a given emission tax  $\tau_e$  and investment subsidy  $\tau_i$ . From Eq. (33) we see that the future welfare gains of a marginal unit of capital (right-hand side) have to equal its current welfare losses due to the costs of labor minus the investment subsidy (left-hand side). Intuitively, the welfare costs of a marginal unit of capital are the lower, the higher is the environmental tax  $\tau_e$  and the higher (i.e., the more negative) is the investment subsidy  $\tau_i$ .

Comparing Eq. (33) with the corresponding condition (14) at the social optimum, we see that, in general, the decentralized market equilibrium falls short of the social optimum, as the welfare costs of investment in the former exceed the corresponding costs in the latter. As a consequence, in the decentralized market solution without policy intervention less favorable circumstances for investment in the new production technique prevail than in the social optimum. To derive the optimal levels for the emission tax and the investment subsidy, we first compare (30) with the corresponding condition (12) at the social optimum. We derive for the optimal emission tax  $\tau_e^{opt}$ :

$$\tau_e(t)^{opt} = \frac{D'(e(t))}{U'(x(t))}. \quad (34)$$

Second, inserting Eq. (24) into Eq. (33) and Eq. (13) into Eq. (14) and comparing the resulting conditions, we derive for the optimal investment subsidy  $\tau_i^{opt}$ :

$$\begin{aligned} \tau_i(t)^{opt} = & - \frac{\exp[-\gamma(t+\sigma)]}{U'(x(t))} \int_{t+\sigma}^{\infty} \frac{U'(x(s))(1 + a(s) - \lambda) + D'(e(s))\lambda (1 - G(a(s)))}{1 + a(s)} \\ & \times \exp[-\gamma s] (\exp[-\rho(s-t)] - \exp[-\rho_p(s-t)]) ds. \end{aligned} \quad (35)$$

If the two instruments are set in such a way that the market equilibrium is identical to the social optimum,  $\tau_e^{opt}$  is always positive (i.e., emissions are taxed) and  $\tau_i^{opt}$  is always negative (i.e., investment is subsidized). The emission tax  $\tau_e^{opt}$  fully internalizes the emission externality, while the investment subsidy  $\tau_i^{opt}$  corrects for the underinvestment in capital due to the split time preference rates between the individual household and the social planner.

#### 4.4 Conditions for Investment and Replacement

In the following, we derive conditions for investment and full replacement in the decentralized economy. Again, we first derive conditions for which the no investment corner solution is a market equilibrium.

**Proposition 3** (No investment condition in the market equilibrium) *Given the household's problem (20), the profit functions (25) and (28) of firm 1 and firm 2, and the emission tax  $\tau_e(t)$  and the investment subsidy  $\tau_i(t)$ , there is no investment in the new technology in the market equilibrium, i.e.,  $i(t) = 0 \forall t$  if and only if the following conditions hold:*

- for  $\tau_e^0 = 0$  (implying  $a^0 = 0$ )

$$1 \leq \lambda + (\gamma + \rho_p) \exp[\rho_p \sigma], \quad (36a)$$

- for  $\tau_e^0 > 0$  (implying  $a^0 > 0$ )

$$1 + a^0 + \frac{1 - G(a^0)}{G'(a^0)} \leq \lambda + \left[ 1 + \frac{\tau_i^0}{\tau_e^0 G'(a^0)} \right] (\gamma + \rho_p) \exp[\rho_p \sigma], \quad (36b)$$

where  $\tau_e^0 = \tau_e(t)$ ,  $\tau_i^0 = \tau_i(t)$  evaluated at the no investment stationary state and, if  $\tau_e^0 > 0$ ,  $a^0$  is determined by the unique solution of the implicit equation:

$$1 = \tau_e^0 [G'(a^0)(1 + a^0) + 1 - G(a^0)]. \quad (37)$$

Condition (36b) for the market equilibrium is identical to the corresponding condition for the social optimum (15), if  $\tau_e^0$  and  $\tau_i^0$  are set as follows:

$$\tau_e^0 = \frac{D'(e^0)}{U'(x^0)} > 0, \quad (38a)$$

$$\tau_i^0 = \frac{D'(e^0)[(1+a^0-\lambda)G'(a^0)+1-G(a^0)]}{U'(x^0)} \left( \frac{\exp[-\rho_p \sigma]}{\gamma + \rho_p} - \frac{\exp[-\rho \sigma]}{\gamma + \rho} \right) < 0, \quad (38b)$$

where  $x^0 = 1 - a^0$  and  $e^0 = (1 - a^0)(1 - G(a^0))$ .

The proof is given in Appendix A.5.

Conditions (36a) and (36b) display the unit costs of energy production of the established and the new technology in the competitive market equilibrium. No investment is a market equilibrium if the established technology displays lower unit costs than the new technology. In the unregulated market regime, the social costs of pollution are not accounted for implying that firm 1 has no incentive to abate.  $UC_{T_1}$  reduces to the 'pure' costs of production, and is, thus, lower than socially optimal.  $UC_{T_2}$  displays the same composition as at the social optimum. However, as it now depends on  $\rho_p > \rho$ , it exceeds the socially optimal unit costs of energy of the new technology. Thus, in the unregulated market economy higher  $UC_{T_2}$  have to stay below lower  $UC_{T_1}$  as compared to the social optimum for the new technology to be innovated. The new technology is disadvantaged in a twofold manner.

Imposing  $\tau_e^0$  enforces the incorporation of the social costs of emissions into the unit costs of production of the established technology. Setting  $\tau_e^0$  equal to the ratio between marginal damage from environmental degradation and marginal benefit from consumption raises  $UC_{T_1}$  to its socially optimal level. However, as is obvious from condition (36b), the imposition of the emission tax is not sufficient for the market equilibrium to resemble the social optimum. Lowering  $UC_{T_2}$  to its socially optimal level can be achieved by payment of an investment subsidy  $\tau_i^0$ .

We now derive the conditions for which full replacement of the established by the new technology is a market equilibrium in the long run.

**Proposition 4** (Full replacement in the market equilibrium) *Given the household's problem (20), the profit functions (25) and (28) of firm 1 and firm 2, the emission tax  $\tau_e(t)$  and the investment subsidy  $\tau_i(t)$ , full replacement of the established technology by the new one in the long-run stationary state is consistent with the necessary and sufficient conditions for a regulated market equilibrium, if and only if the following condition holds:*

- for  $\tau_e^0 = 0$  (implying  $a^0 = 0$ )

$$1 \geq \lambda + (\gamma + \rho_p) \exp[\rho_p \sigma]. \quad (39a)$$

- for  $\tau_e^0 > 0$  (implying  $a^0 > 0$ )

$$1 + \frac{\tau_e^\infty}{1 - \tau_e^\infty} \geq \lambda + \left[ 1 + \frac{\tau_i^\infty}{1 - \tau_e^\infty} \right] (\gamma + \rho_p) \exp[\rho_p \sigma], \quad (39b)$$

where  $\tau_e^\infty = \tau_e(t)$ ,  $\tau_i^\infty = \tau_i(t)$  evaluated at the long-run stationary state.

Condition (39b) for the market equilibrium is identical to the corresponding condition for the social optimum (17), if  $\tau_e^\infty$  and  $\tau_i^\infty$  are set as follows:

$$\tau_e^\infty = \frac{D'(0)}{U'(x^\infty)} \geq 0, \quad (40)$$

$$\tau_i^\infty = \frac{U'(x^\infty)(1 - \lambda) + D'(0)\lambda}{U'(x^\infty)} \left( \frac{\exp[-\rho_p \sigma]}{\gamma + \rho_p} - \frac{\exp[-\rho \sigma]}{\gamma + \rho} \right) < 0, \quad (41)$$

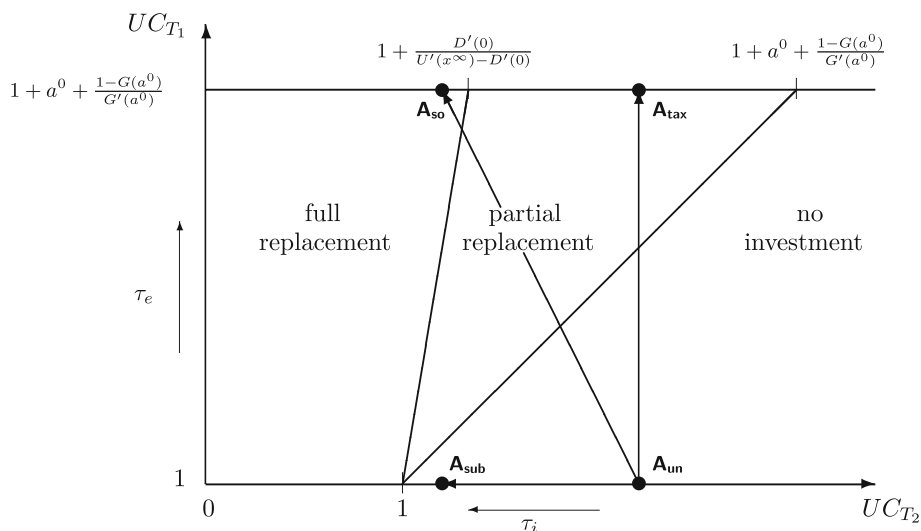
where  $x^\infty = \frac{1}{\lambda + \gamma}$ .

The proof is given in Appendix A.6

The economic interpretation of conditions (39a) and (39b) is analogous to that of conditions (36a) and (36b). Although the external effect from the emissions vanishes in case the new technology fully replaces the old one, a positive emission tax has to be raised if  $D'(0) > 0$  for the market equilibrium to resemble the social optimum. For  $D'(0) = 0$ , the optimal tax in the full replacement stationary state is given by  $\tau_e^\infty = 0$ . The optimal investment subsidy  $\tau_i^\infty$  has to be negative in any case. For full replacement to occur in the regulated market regime in the long run, condition (36b) has to be violated while condition (39b) holds. However, if  $\tau_e$  and  $\tau_i$  are such that both conditions (36b) and (39b) are simultaneously violated, the economy exhibits a market equilibrium where both technologies are used. That is, partial replacement of the established by the new technique is optimal. Note that partial replacement of the established technology by the new one cannot occur in the unregulated market regime.

## 5 Discussion

Before discussing model assumptions and policy implications, we briefly summarize the findings of our analysis. Recall that there are two energy technologies available in the economy. The first gives rise to emissions which can be partly abated by an end-of-pipe technology. The resulting net emissions impose a negative externality on society. The second is clean but needs some time  $\sigma$  before investment becomes productive. Moreover, the intertemporal valuation is deterred by the split between the private and social rates of time preference. Whether the second technology (partly) replaces the first one hinges on the exogenously



**Fig. 1** Full replacement, partial replacement, and no investment in the unregulated market equilibrium and the social optimum

given parameters and on whether and to what extent the emission externality and the split of time preferences are corrected by an emission tax  $\tau_e$  and an investment subsidy  $\tau_i$ . Figure 1 illustrates the findings.

In the unregulated market regime  $UCT_1$  always equals 1. Thus, the combination of the unit costs of production of the two technologies associated with the investment and replacement conditions is always represented by a point on the  $UCT_2$  axis in Fig. 1. For example, point  $A_{un}$  denotes a situation where no investment in the new technology takes place in the unregulated market regime, though full replacement would be socially optimal (point  $A_{so}$ ). Imposing an emission tax  $\tau_e$  increases  $UCT_1$  (upwards shift in Fig. 1). At the social optimum  $UCT_1$  equals  $1 + a^0 + \frac{1-G(a^0)}{G'(a^0)}$ . The introduction of an investment subsidy decreases  $UCT_2$ , shifting it to the left in Fig. 1. In general, the social optimum in a market regime can only be implemented by combining environmental and technology policies (moving from  $A_{un}$  to  $A_{so}$ ). In the example, the sole imposition of the emission tax would lead to a partial replacement of the established technology (shift from  $A_{un}$  to  $A_{tax}$ ), and the sole imposition of the investment subsidy leaves the economy in the no-investment stationary state (shift from  $A_{un}$  to  $A_{sub}$ ).

### 5.1 Model Assumptions

In our analysis, we explore the welfare-theoretic implications of diverging social and private time preferences for the time-lagged transition from a polluting established to a new clean technology. Although our model considers important features of the energy industry, we make a series of simplifying assumptions, which we shall briefly discuss in the following.

In our model we consider a flow pollutant, whereas the accumulation of greenhouse gases in the atmosphere causing the rise of global mean temperature is a stock-pollutant problem. This simplification does not qualitatively affect our results. However, for a stock pollutant, the split of time-preference rates would imply an underestimation of the future damages from emissions today by the individual households compared to the social planner. As a



consequence, the unit costs of production of the established technology would be further underestimated in the not optimally regulated market economy.

By modeling the technologies as linear and linear-limitational, we assume very specific functional forms. The rationale is to account for the rigidities in energy conversion due to technical and thermodynamic constraints. From a more technical point of view, it is the linearity of the production functions which gives rise to the corner solutions we exploit to derive the conditions of investment and partial and full replacement. As our focus is on the substitution effects *between* (the established and the new) production technologies, the analysis abstracts from substitution possibilities among different production factors *within* the individual technologies. Taking a medium-term perspective with invariable technologies, we abstract furthermore from some typical long-run problems. First, we neglect endogenous technological change in the sense that new technologies emerge or technologies become more efficient over time. Second, we do not consider fuel inputs explicitly and, thus, implicitly assume the finiteness of conventional energy sources to be non-binding over the relevant time horizon. Finally, we abstract from growth. Obviously, all these characteristics are important for successful climate-change mitigation strategies but are not in the primary focus of our paper.

Finally, for the sake of a tractable model we abstract from a series of peculiarities relevant in the economics of electric power systems. First, the energy industry is subject to cyclical demand fluctuations on different time-scales (for example day/night-time or summer/winter). As different energy technologies exhibit different turn-on/turn-off costs and rigidities, a mix of energy technologies is in general preferable over ‘energy monocultures’. Second, in contrast to our assumption of a perfectly competitive market, the energy industry rather exhibits an oligopolistic market structure. As is well known from the industrial organization literature, unregulated oligopolistic market regimes lead in general to additional market failures, from which we abstract to concentrate on the distortions imposed by emissions and diverging time-preference rates.

## 5.2 Policy Implications

Although the analysis has been carried out in a stylized theoretical framework, direct policy implications can be drawn which are relevant for the regulation of the energy industry in particular, and optimal technological transitions in general.

First, our analysis shows that the time lag in the production of capital amplifies the distortion created by the split of time preference rates. In Table 1 we illustrate the effect of time-to-build and split time preference rates on the unit costs of the new technology for different values of the labor costs  $\lambda$  and time lags  $\sigma$ .<sup>10</sup> The first number displays the steady-state unit costs in the social optimum,  $\lambda + (\gamma + \rho) \exp[\rho\sigma]$ , the second number the steady-state unit costs without investment subsidy,  $\lambda + (\gamma + \rho_p) \exp[\rho_p\sigma]$ , and the third number the difference in percent of the unit costs in the social optimum. We see that without correcting for the split time preference rates the new production technology is particularly disadvantaged (as a comparison to its unit costs in the social optimum) if pure labor costs are small implying that the share of capital costs for total unit costs is high, and if the gestation lag  $\sigma$  is large. For example, for  $\lambda = 0.1$  which implies that capital costs amount to 50% of total unit costs and a time lag  $\sigma = 6$  years, a difference in time preference rates of  $\rho = 0.03$  per year

<sup>10</sup> The following parameters have been chosen:  $\rho = 0.03$ ,  $\rho_p = 0.07$ ,  $\gamma = 0.05$  (all rates per year). While the calculations are relatively insensitive to changes in  $\gamma$ , they are very sensitive to the split in time preference rates  $\rho_p - \rho$ .

**Table 1** Calculation of steady-state unit costs of the new technology in the social optimum (first number) and in the market economy without investment subsidy (second number) for different gestation lags  $\sigma$  and pure labor costs  $\lambda$ 

$\lambda$	0.1	0.3	0.5	0.7	0.9
$\sigma = 0$	0.18, 0.22, 22.2%	0.38, 0.42, 10.5%	0.58, 0.62, 6.9%	0.78, 0.82, 5.1%	0.98, 1.02, 4.1%
$\sigma = 2$	0.18, 0.24, 28.7%	0.38, 0.44, 13.8%	0.58, 0.64, 9.1%	0.78, 0.84, 6.8%	0.98, 1.04, 5.4%
$\sigma = 4$	0.19, 0.26, 36.1%	0.39, 0.46, 17.6%	0.59, 0.66, 11.6%	0.79, 0.86, 8.7%	0.99, 1.06, 6.9%
$\sigma = 6$	0.20, 0.28, 44.4%	0.40, 0.48, 21.9%	0.60, 0.68, 14.6%	0.80, 0.88, 10.9%	1.00, 1.08, 8.7%
$\sigma = 8$	0.20, 0.31, 53.7%	0.40, 0.51, 27.0%	0.60, 0.71, 18.0%	0.80, 0.91, 13.5%	1.00, 1.11, 10.8%
$\sigma = 10$	0.21, 0.34, 64.3%	0.41, 0.54, 32.8%	0.61, 0.74, 22.0%	0.81, 0.94, 16.5%	1.01, 1.14, 13.3%

The third number states the difference in percent of the unit costs of the social optimum

and  $\rho_p = 0.07$  per year results in an overestimation of the unit costs by 44.4% compared to the unit costs in the social optimum. As the energy sector exhibits both a high share of capital costs and substantial construction lags, we expect it to be particularly affected. This expectation is confirmed by [Heinzel \(2008\)](#), who conducts an analysis of the German power industry around 2015, which shows that the distortion induced on the (imputed) unit costs of electricity at the busbar of new coal, gas or nuclear power plants may amount to 1.0–18.5 €/MWh. In addition, its elimination may have a decisive impact on the technology ranking.

Second, our analysis gives new theoretical support for policies that subsidize the deployment of energy technologies. According to our analysis, the level of the subsidy should, in particular, depend on the difference in time-preference rates and the time lag in construction of the new technology. Thus, we are rather skeptical about the efficiency of policies such as the German “Erneuerbare-Energien-Gesetz” (Renewable Energy Sources Act) that subsidizes electricity from renewable energy technologies by feed-in tariffs oriented at the level of their unit costs of production.<sup>11</sup>

Finally, the analysis implies that for the transition towards a low-emission energy industry the imposition of an environmental tax alone is in general not sufficient to implement the socially optimal path.<sup>12</sup> Rather, technology policy should complement environmental policy. As a general result, this is not new, as there is a series of well established causes for technology policy associated with the process of technological transformation (e.g., [Jaffe et al. 2005](#)). We derive this result without considering these cases. In our model, it is the split of social and private time-preference rates combined with the time-consuming nature of bringing a new technology into use which leads to the additional distortion. Thus, the split time-preference rates constitute a general case for a welfare-enhancing policy intervention, irrespective of the causes of the split. However, the intervention is to be directed towards the source of the distortion. The split of the rates itself may only be the direct point of reference, if and only if the underlying market failure cannot directly or differently be remedied.

## 6 Conclusion

We study the implications of diverging social and private time-preference rates for the transition from an established polluting to a new clean energy technology in a time-lagged general

<sup>11</sup> Feed-in tariffs amount to 457–624 €/MWh for photovoltaics, 55–91 for wind energy and 71.6–150 for geothermal energy, which is way off the 1.0–18.5 €/MWh computed by [Heinzel \(2008\)](#) for the distortion induced by the split time-preference rates.

<sup>12</sup> The equivalent result holds for the sole introduction of an emission permit trading scheme.

equilibrium model. The two distortions in the model create in a mutually reinforcing way less favorable circumstances for the introduction of the new technology, and hence delay or even hinder structural change as compared to the social optimum.

The distortion created by the split of time-preference rates and amplified by time-to-build feature of capital constitutes a general case for a welfare-enhancing policy intervention, irrespective of the causes of the split. The split of the rates itself may, however, be the direct point of reference, if and only if the underlying market failure cannot directly or differently be remedied. We show for this case that the socially optimal path may be implemented if, in addition to standard environmental policy, an investment subsidy is paid. Our results constructively contribute to the questions of whether and how environmental policy should be complemented by further measures, such as technology policy.

In different respects, our analysis sticks to simplest cases. In particular, we avoid an endogenous explanation of the split of the rates in the model and consider a flow pollutant. While this is sufficient to clarify the basic relationships, it points to a number of issues for further research. Thus far, there has been no systematic analysis of the causes of the split of social and private time preferences, their quantitative contribution to the split, and the specific policy implications with respect to each cause. Further theoretical investigations should especially account for a richer representation of preferences.

## Appendix A

### A.1 Concavity of the Hamiltonian (10)

We show that the Hamiltonian (10) is strictly and jointly concave in  $a$ ,  $i$  and  $k$  whenever the necessary condition (12) holds. We first introduce function  $F(a(t), i(t), k(t))$  defined as

$$F(a(t), i(t), k(t)) = U\left(\frac{1 - \lambda k(t) - i(t)}{1 + a(t)} + k(t)\right) - D\left((1 - G(a(t))) \frac{1 - \lambda k(t) - i(t)}{1 + a(t)}\right). \quad (\text{A.1})$$

Due to the linearity of the equation of motion (5) and the inequality conditions (9b), it is sufficient to show that  $F$  is strictly and jointly concave in  $a$ ,  $i$  and  $k$  whenever (12) holds.<sup>13</sup>  $F$  is strictly concave if the determinants of the leading principal minors of the Hessian  $H$  of  $F$  are alternating in sign, starting with a negative sign. This is equivalent to  $H$  being negative definite.

Denoting by  $F_{yz}^0$  the second partial derivatives of  $F$  with respect to  $y$  and  $z$ , given that the necessary condition (12) holds, we obtain:

$$F_{aa}^0 = \frac{1 - \lambda k - i}{(1 + a)^2} \left\{ \frac{1 - \lambda k - i}{(1 + a)^2} [U''(x) - D''(e)[1 - G(a) + (1 + a)G'(a)]^2 \right. \\ \left. + D'(e)G''(a) \right\}, \quad (\text{A.2a})$$

$$F_{ai}^0 = \frac{1 - \lambda k - i}{(1 + a)^3} \{U''(x) - D''(e)[1 - G(a)][1 - G(a) + (1 + a)G'(a)]\}, \quad (\text{A.2b})$$

<sup>13</sup> In the following, we refrain from stating the time argument explicitly.

$$F_{ak}^0 = -\frac{1-\lambda k-i}{(1+a)^3} \left\{ U''(x)(1-\lambda+a) + \lambda D''(e) \right. \\ \left. \times [1-G(a)][1-G(a) + (1+a)G'(a)] \right\}, \quad (\text{A.2c})$$

$$F_{ii}^0 = \frac{1}{(1+a)^2} \left\{ U''(x) - D''(e)[1-G(a)]^2 \right\}, \quad (\text{A.2d})$$

$$F_{ik}^0 = -\frac{1}{(1+a)^2} \left\{ U''(x)(1-\lambda+a) + \lambda D''(e)[1-G(a)]^2 \right\}, \quad (\text{A.2e})$$

$$F_{kk}^0 = \frac{1}{(1+a)^2} \left\{ U''(x)(1-\lambda+a)^2 - \lambda^2 D''(e)[1-G(a)]^2 \right\}. \quad (\text{A.2f})$$

Calculating the determinants of the leading principal minors of the Hessian  $H$

$$\det[H^1] = F_{aa}^0 < 0, \quad (\text{A.3a})$$

$$\det[H^2] = F_{aa}^0 F_{ii}^0 - (F_{ai}^0)^2 > 0, \quad (\text{A.3b})$$

$$\det[H^3] = \det[H] = -\frac{1-\lambda k-i}{(1+a)^4} U''(x) D'(e) D''(e) G''(a) [1-G(a)]^2 < 0, \quad (\text{A.3c})$$

reveals that  $H$  is negative definite.

## A.2 Optimal System Dynamics

In the following, we discuss the optimal system dynamics of the optimization problem (9) in case of the first and second regime:

- (i) In the no investment regime,  $i(t) = 0 \forall t$  holds. As a consequence, also  $k(t) = 0 \forall t$ , and the system remains in a stationary state where the labor endowment is fully used up by energy production via the established technology and by abatement:  $x^0 = x_1^0 = 1 - a^0$ ,  $e^0 = (1 - a^0)(1 - G(a^0))$ , and  $a^0$  is given implicitly by Eq. (12), which yields a unique solution as shown in the proof of Proposition 1.
- (ii) If the new technology eventually fully replaces the established technology, then all labor is used to employ and maintain the capital stock  $k$ . Thus,  $a(t) = l_1(t) = 0$ , and we derive from the labor constraint (6):

$$k(t) = \frac{1-i(t)}{\lambda}, \quad (\text{A.4})$$

which expresses capital  $k(t)$  in terms of investment  $i(t)$ . Differentiating with respect to time  $t$  and inserting into the equation of motion for the capital stock (5), yields the following differential-difference equation, which governs the long-run system dynamics:

$$\frac{di(t)}{dt} + \gamma i(t) + \lambda i(t-\sigma) = \gamma. \quad (\text{A.5})$$

According to Theorem 3.3 (Bellman and Cooke 1963, p. 53), the solution is given by the superposition of the solution to the homogeneous equation

$$\frac{di(t)}{dt} + \gamma i(t) + \lambda i(t-\sigma) = 0, \quad (\text{A.6})$$

and a particular solution for the inhomogeneous equation. For  $di(t)/dt = 0$  Eq. (A.5) yields the non-trivial stationary state

$$i^\infty = \frac{\gamma}{\lambda + \gamma}, \quad k^\infty = \frac{1}{\lambda + \gamma}. \quad (\text{A.7})$$

According to Theorem 3.4 (Bellman and Cooke 1963, p. 55) the solution to the homogeneous equation can be written as

$$\sum_n p_n(t) \exp[y_n t], \quad (\text{A.8})$$

where  $y_n$  are the roots of the characteristic equation

$$h(y) \equiv y + \gamma + \frac{\lambda}{\gamma} \exp[-\sigma y], \quad (\text{A.9})$$

and  $p_n(t)$  is a polynomial in  $t$  of degree less than the multiplicity of the characteristic root  $y_n$ .<sup>14</sup> Thus, the general solution is given by

$$i(t) = i^\infty + \sum_n p_n(t) \exp[y_n t]. \quad (\text{A.10})$$

It can be shown that the characteristic equation (A.9) has at most two negative real roots and an infinite number of conjugate pairs of complex roots of which only a finite number have positive real part (Proposition 2 in Winkler et al. 2005). All summands which correspond to characteristic roots with negative real parts converge to zero in the long-run. There may be one summand corresponding to a pair of purely imaginary roots (which, of course, then collapse to one root), which oscillates around 0.<sup>15</sup> All summands corresponding to characteristic roots with positive real part are diverging oscillatory for  $t \rightarrow \infty$ .

While diverging summands are solutions to the differential-difference equation (A.5), they are no solutions to the system dynamics of the second regime. This is because investment is bounded to 1 by the labor constraint. All diverging solutions are not feasible, as they eventually violate the labor constraint. This implies directly that also the capital stock cannot diverge in the second regime. Moreover, all diverging solutions do not only violate the labor constraint, but also the non-negativity constraint  $i(t) \geq 0$ . To see this, consider the pair of summands corresponding to the pair of complex conjugate roots with the highest positive real part, say  $y = a \pm ib$ ,  $a, b > 0$ . These two summands can be written as:

$$K_1 \exp[at] \cos[bt + K_2], \quad (\text{A.11})$$

with two real constants  $K_1$  and  $K_2$ . As the cosine is at times positive and at times negative, we have a divergent oscillation, which implies that investment not only diverges over time but also switches from periods in which it is positive to periods in which it is negative. In most economic models, negative investment and capital stocks have no meaningful interpretation.

In summary, the optimal solution converges to the stationary state if there exists no complex root with vanishing real part, and to a limit cycle around the stationary state otherwise. As the latter case can only hold accidentally for certain exogenous parameter constellations, we restrict attention to the case of convergence to the stationary state (A.7).

### A.3 Proof of Proposition 1

Assume that it is optimal not to invest at  $t = 0$ , which implies that it is optimal not to invest at all times  $t$ . As a consequence, the economy will remain in the no investment corner

<sup>14</sup> It is easy to verify that there is at most one multiple root corresponding to  $y = -(\gamma + 1/\sigma)$  which only occurs when  $\lambda = (\gamma/\sigma) \exp[-(1 + \sigma\gamma)]$ .

<sup>15</sup> For this to hold, there has to exist  $b \in [0, \lambda]$  which simultaneously solves  $\gamma = -\lambda \cos[\sigma b]$  and  $b = \lambda \sin[\sigma b]$ .

solution where no capital is accumulated. Hence,  $i(t) = 0$  and  $q_i(t) \geq 0 \forall t$ . All energy is solely produced by the established production technique which implies that  $x^0 = x_1^0 = l_1^0 = 1 - a^0$ ,  $x_2^0 = 0$ ,  $> 0$  and  $q_i^0 = 0$ . The optimal abatement effort  $a^0$  is determined by Eq. (12) by inserting  $x^0 = 1 - a^0$  and  $e^0 = x^0(1 - G(a^0))$  which yields Eq. (16). To see that there exists a unique solution for  $a^0$  we define the following functions which correspond to the left-hand side and the right-hand side of Eq. (16):

$$lhs(a^0) = U'(1 - a^0), \quad (\text{A.12a})$$

$$rhs(a^0) = D'((1 - a^0)(1 - G(a^0))) [1 - G(a^0) + (1 + a^0)G'(a^0)]. \quad (\text{A.12b})$$

Uniqueness is guaranteed by  $lhs'(a^0) > 0$  and  $rhs'(a^0) < 0$ . Existence holds as

$$\begin{aligned} \lim_{a^0 \rightarrow 0} lhs(a^0) &= U'(1) \in (0, \infty), & \lim_{a^0 \rightarrow 1} lhs(a^0) &= +\infty, \\ \lim_{a^0 \rightarrow 0} rhs(a^0) &= +\infty, & \lim_{a^0 \rightarrow 1} rhs(a^0) &= 0. \end{aligned}$$

In the corner solution  $i(t) = 0$ , we derive the shadow price of capital  $q_k^0(t)$  by solving the integral Eq. (13):

$$q_k^0(t) = D'(e^0) [(1 + a^0 - \lambda) G'(a^0) + 1 - G(a^0)] \frac{\exp[-\rho t]}{\gamma + \rho}. \quad (\text{A.13})$$

Inserting Eq. (12) and  $q_k^0(t + \sigma)$  into (14) yields the following necessary and sufficient condition for the corner solution to be optimal:

$$\begin{aligned} D'(e^0) G'(a^0) \exp[-\rho t] &= D'(e^0) [(1 + a^0 - \lambda) G'(a^0) + 1 - G(a^0)] \frac{\exp[-\rho(t + \sigma)]}{\gamma + \rho} \\ &+ q_i(t). \end{aligned} \quad (\text{A.14})$$

Taking into account that  $q_i(t) \geq 0$ , dividing by  $D'(e^0) G'(a^0) \exp[-\rho t]$  and rearranging terms yields condition (15). Note that condition (15) is independent of  $t$ . This implies that it is optimal not to invest at all times  $t$ , if it is optimal not to invest at time  $t = 0$ . Thus, if condition (15) holds, the optimal solution of the optimization problem (9) is to remain in the no investment corner solution forever.

#### A.4 Proof of Proposition 2

Assume that it is optimal in the long-run stationary state to use the total labor endowment to employ and maintain the capital stock for the new technology, i.e.,  $x_2^\infty = \frac{1}{\lambda + \gamma}$ . Then, all output is solely produced by the new technology, i.e.,  $x^\infty = x_2^\infty$ ,  $x_1^\infty = l_1^\infty = 0$ . In addition,

no emissions are produced and have to be abated implying  $e^\infty = 0$  and  $a^\infty = 0$ . Solving the integral Eq. (13) yields:

$$q_k^\infty(t) = \frac{\exp[-\rho t]}{\gamma + \rho} \left[ U'(x^\infty)(1 - \lambda) + \lambda \left( D'(0) - q_i^\infty \right) \right]. \quad (\text{A.15})$$

Inserting  $q_k^\infty(t + \sigma)$  into Eq. (14), and taking into account that  $q_i^\infty \geq 0$ , we derive condition (17).  $\square$

### A.5 Proof of Proposition 3

Assume that no investment at all times  $t$  is a market equilibrium. Then, no capital is accumulated and  $i(t) = 0$  and  $q_i(t) \geq 0 \forall t$ . All energy is solely produced by the established production technique (i.e.,  $x^0 = x_1^0 = l_1^0 = 1 - a^0$ ,  $x_2^0 = 0$ ). We know from conditions (27) and (29):

$$w(t) = \frac{1 - \tau_e^0 (1 - G(a^0))}{1 + a^0}, \quad (\text{A.16a})$$

$$r(t) > \frac{1 + a^0 - \lambda [1 - \tau_e^0 (1 - G(a^0))]}{1 + a^0}. \quad (\text{A.16b})$$

Equation (A.20) determines the profit maximizing abatement effort  $a^0$  of firm 1 if  $\tau_e^0 > 0$  (otherwise the optimal  $a^0 = 0$ ). Inserting condition (A.16b) in Eq. (23b) and solving the differential equation, yields the following inequality for the shadow price of capital:

$$q_k^0(t) \geq \frac{1 + a^0 - \lambda [1 - \tau_e^0 (1 - G(a^0))]}{(1 + a^0)(\gamma + \rho_p)} U'(x^0) \exp[-\rho_p t]. \quad (\text{A.17})$$

Inserting Eq. (A.16a) and  $q_k^0$  into Eq. (23a), and taking into account that  $q_i(t) \geq 0$ , we derive:

$$(w(t) + \tau_i^0) U'(x^0) \exp[-\rho_p t] \geq \frac{1 + a^0 - \lambda [1 - \tau_e^0 (1 - G(a^0))]}{(1 + a^0)(\gamma + \rho_p)}. \quad (\text{A.18})$$

Dividing by  $U'(x^0) \exp[-\rho_p t]$  and rearranging terms yields that in the regulated market equilibrium there is no investment in the new technology, if and only if:

$$\frac{1 + a^0}{1 - \tau_e^0 (1 - G(a^0))} \leq \lambda + \left[ 1 + \frac{\tau_i^0 (1 + a^0)}{1 - \tau_e^0 (1 - G(a^0))} \right] (\gamma + \rho_p) \exp[\rho_p \sigma] \quad (\text{A.19})$$

If  $a^0 = \tau_e^0 = \tau_i^0 = 0$ , condition (A.19) reduces to (36a). If  $\tau_e^0 > 0$ , Eq. (26) holds and  $a^0$  is given by the following implicit equation:

$$1 = \tau_e^0 [G'(a^0)(1 + a^0) + 1 - G(a^0)]. \quad (\text{A.20})$$

For an exogenously given  $\tau_e^0$  the right-hand side of Eq. (A.20) is strictly decreasing. Moreover, it approaches  $+\infty$  for  $a^0 \rightarrow 0$  and 0 for  $a^0 \rightarrow 1$ . This implies that there exists a unique solution for  $a^0$  whenever  $\tau_e^0 > 0$ . Inserting into condition (A.19) yields (36b).

By setting  $\tau_e^0 = D'(e^0)/U'(x^0)$ , condition (A.20) which determines the profit maximizing abatement effort  $a^0$  becomes identical to Eq. (16) which determines the socially optimal abatement level. Again, there exists a unique solution for  $a^0$  as shown in the proof of Proposition 1. Furthermore, inserting  $\tau_e^0$  and  $\tau_i^0$  from Eq. (38a) and (38b) into condition

(36b) yields (after some tedious calculations) the no investment condition in the social optimum (15).

#### A.6 Proof of Proposition 4

Assume that using the total labor endowment to employ and maintain the capital stock for the new technology in the long-run stationary state is a market equilibrium, i.e.,  $l_1^\infty = 0$ ,  $i^\infty > 0$  and  $q_i^\infty = 0$ . Then, all output is solely produced by the new technology, i.e.,  $x^\infty = x_2^\infty = \frac{1}{\lambda + \gamma}$  and  $x_1^\infty = l_1^\infty = 0$ . In addition, no emissions are produced and have to be abated and, thus,  $e^\infty = 0$  and  $a^\infty = 0$ . For this case, we know from demand correspondences (27) and (29) of firm 1 and firm 2:

$$w(t) \leq 1 - \tau_e(t), \quad (\text{A.21a})$$

$$r(t) = 1 - \lambda w(t). \quad (\text{A.21b})$$

Inserting Eq. (A.21b) into Eq. (24), yields for the the shadow price of capital:

$$q_k^\infty(t) = \frac{1 - \lambda w^\infty}{\gamma + \rho_p} U'(x^\infty) \exp[-\rho_p t], \quad (\text{A.22})$$

where  $w^\infty = w(t)$  is evaluated at the full replacement stationary state and, hence, constant.

Inserting  $q_k^\infty$  and inequality (A.21a) into Eq. (23a), and taking into account that  $q_i(t) = 0$ , we derive the following condition:

$$(1 - \tau_e^\infty) (\lambda + (\gamma + \rho_p) \exp[\rho_p \sigma]) \leq 1 - \tau_i^\infty (\gamma + \rho_p) \exp[\rho_p \sigma]. \quad (\text{A.23})$$

Dividing by  $(1 - \tau_e^\infty)$  and rearranging terms yields condition (39b). Setting  $\tau_e^\infty = \tau_i^\infty = 0$ , we derive condition (39a).

Furthermore, inserting  $\tau_e^\infty$  and  $\tau_i^\infty$  from Eqs. (40) and (41) into condition (39b) yields (after some tedious calculations) the full replacement condition in the social optimum (17).  $\square$

## References

- Arrow KJ, Kurz M (1970) Public investment, the rate of return, and optimal fiscal policy. John Hopkins, Baltimore
- Arrow KJ, Lind RC (1970) Uncertainty and the evaluation of public investment decisions. *Am Econ Rev* 60(3):364–378
- Asea PK, Zak PJ (1999) Time-to-build and cycles. *J Econ Dyn Control* 23(8):1155–1175
- Bambi M (2008) Endogenous growth and time-to-build: the AK case. *J Econ Dyn Control* 32(4):1015–1040
- Bellman R, Cooke KL (1963) Differential-difference equations. Academic Press, New York
- Blanchard OJ (1985) Debt, deficits, and finite horizons. *J Polit Econ* 93(2):223–247
- Boucekkine R, Licandro O, Puch LA, del Rio F (2005) Vintage capital and the dynamics of the AK model. *J Econ Theory* 120(1):39–72
- Bovenberg AL, Smulders S (1995) Environmental quality and pollution-augmenting technological change in a two-sector endogenous growth model. *J Public Econ* 57(3):369–391
- Calvo GA, Obstfeld M (1988) Optimal time-consistent fiscal policy with finite lifetimes. *Econometrica* 56(2):411–432
- DeMarzo P, Fishman M, He Z, Wang N (2009) Dynamic agency and the  $q$  theory of investment. Mimeo
- Drèze J, Stern N (1990) Policy reform, shadow prices, and market prices. *J Public Econ* 42(1):1–45
- El-Hodiri MA, Loehman E, Whinston A (1972) An optimal growth model with time lags. *Econometrica* 40(6):1137–1146
- Fabbri G, Gozzi F (2008) Solving optimal growth models with vintage capital: the dynamic programming approach. *J Econ Theory* 143(1):331–373



- Frederick S, Loewenstein G, O'Donoghue T (2002) Time discounting and time preference: a critical review. *J Econ Lit* 40(2):351–401
- Gandolfo G (1996) Economic dynamics, third, completely revised and enlarged edition. Springer, Berlin
- Gerlagh R, Van der Zwaan B (2003) Gross world product and consumption in a global warming model with endogenous technological change. *Resour Energy Econ* 25(1):35–57
- Gollier C (2002) Discounting an uncertain future. *J Public Econ* 85(2):149–166
- Gollier C (2010) Pricing the future: the economics of discounting and sustainable development. Unpublished manuscript, Princeton University Press, Princeton (to appear)
- Goulder LH, Mathai K (2000) Optimal CO<sub>2</sub> abatement in the presence of induced technological change. *J Environ Econ Manage* 39(1):1–38
- Goulder LH, Schneider SH (1999) Induced technological change and the attractiveness of CO<sub>2</sub> abatement policies. *Resour Energy Econ* 21(3–4):211–253
- Groom B, Hepburn C, Koundouri P, Pearce D (2005) Declining discount rates: the long and the short of it. *Environ Resour Econ* 32(1):445–493
- Hale J (1977) Theory of functional differential equations. Springer, New York
- Heinzel C (2008) Implications of diverging social and private discount rates for investments in new generation technology. A New Case for Nuclear Power? Paper presented at the annual meeting of the Verein für Socialpolitik in Graz, Austria
- Jaffe AB, Newell RG, Stavins RN (2005) A tale of two market failures: technology and environmental policy. *Ecol Econ* 54(2–3):164–174
- Kamien MI, Schwartz NL (1992) Dynamic optimization. The calculus of variations and optimal control in economics and management. 2nd edn. Elsevier, Amsterdam
- Kolmanovskii VB, Myshkis AD (1999) Introduction to the theory and applications of functional differential equations. Kluwer, Dordrecht
- Mehra R, Prescott EC (2003) The equity premium in retrospect. In: Constantinides GM, Harris M, Stulz R (eds) Handbook of the economics of finance. Elsevier B.V, Amsterdam pp 887–936
- Newell RG, Jaffe AB, Stavins RN (1999) The induced innovation hypothesis and energy-saving technological change. *Q J Econ* 114(3):941–975
- Nordhaus WD (2002) Modeling induced innovation in climate-change policy. In: Grubler A, Nakicenovic N, Nordhaus WD (eds) Technological change and the environment. Resources for the Future, Washington, DC pp 182–209
- Portney PR, Weyant JP (eds) (1999) Discounting and intergenerational equity. Resources for the Future, Washington, DC
- Stiglitz JE (1982) The Discount Rate for Benefit-Cost Analysis and the Theory of the Second Best. In: Lind RC, Arrow KJ, Corey GR, Dasgupta P, Sen AK, Stauffer T, Stiglitz JE, Stockfish JA, Wilson R (eds) Discounting for time and risk in energy policy. John Hopkins University Press, Baltimore pp 151–204
- Tahvonen O, Salo S (2001) Economic growth and transition between renewable and nonrenewable energy resources. *Eur Econ Rev* 45(8):1379–1398
- Tirole J (1981) Taux d'Actualisation et optimum second (discount rate and second-best optimum). *Rev Econ* 32(5):829–869
- Van der Zwaan BCC, Gerlagh R, Klaassen G, Schrattenholzer L (2002) Endogenous technological change in climate change modelling. *Energy Econ* 24(1):1–19
- Winkler R (2005) Structural change with joint production of consumption and environmental pollution: a neo-Austrian approach. *Struct Change Econ Dyn* 16(1):111–135
- Winkler R (2008) Optimal compliance with emission constraints: dynamic characteristics and the choice of technique. *Environ Resour Econ* 39(4):411–432
- Winkler R, Brand-Pollmann U, Moslener U, Schlöder J (2005) On the transition from instantaneous to time-lagged capital accumulation. The case of Leontief-type production functions. Discussion-Paper No. 05-30. Centre for European Economic Research (ZEW), Mannheim
- Yaari ME (1965) Uncertain lifetime, life insurance, and the theory of the consumer. *Rev Econ Stud* 32(2): 137–150